MODELS OF UNIFORMLY FLUIDIZED BED OF SPHERICAL PARTICLES FOR Ar $\leq$ 7·2

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Geometrical arrangement of particles in the uniformly fluidized bed and some properties of liquid flow around particles are studied. Steady liquid flow in the uniformly fluidized bed of spherical particles for Ar $\leq$ 7·2 is considered to be a flow around a single particle at boundary conditions which are uniquely corresponding to the bed porosity and to the particle diameter. This approach had been already applied but the required result has not been reached as the theoretical fundamentals of the model have not been sufficiently solved. The basical problem we have succeeded to solve is specification of boundary conditions and development of the mathematical model. By solving the mathematical model and by use of the experiments performed the geometrical arrangement of particles is determined.

Particles in the uniformly fluidized bed are at small values of the Ar number in the state of stable equilibrium. This can be observed visually. If the velocity distribution is uniform at the inlet into the bed and certain other conditions are met, the particles will oscillate slightly around the equilibrium position. According to this the assumption that particles, at small Ar numbers in the uniformly fluidized bed of particles at constant porosity, are occupying the accuratelly fixed positions seems to be reasonable. The hydrodynamics of the uniformly fluidized bed can be considered as the flow around a particle under certain boundary conditions which are uniquely assigned to the bed porosity and to the particle diameter. If the geometrical distribution of particles in the bed and the structure of the flow around the particle were known, the greater would be the chance that the following problems could be solved in the near future: stability of the fluidized bed and criteria of nonuniformity, saturation of the dilute phase at flow of the suspension solid particles-gas, equivalent diameter at flow of the suspension with particles of various sizes, diffusion of components of the liquid in the uniformly fluidized bed, expansion of the uniformly fluidized bed *etc.*

THEORETICAL MODELS

The following assumptions form the basis of the theoretical model.

1) Particles have the shape of ideal sphere, are of the same dimension, have the same density and in the bed at constant porosity have the same position without rotation. *2*) The liquid is Newtonian and incompressible. At the flow in the bed its Newtonian character is not changing and it holds $\varrho_f < \varrho_s$ but both densities do not differ much. *3*) Liquid flow in the bed (upwards) is steady (without turbulent velocity pulsation). *4*) The bed is unlimited, *i.e.* the boundary effects do not appear. *5*) Flow pattern for flow around each particle is the same. *6*) The particles are situated one above the other in layers in horizontal planes so that to each horizontal plane passing through centres of particles (*i.e.* the main plane) to each particle pertains geometrically the same environment formed by other particles (so called boundary surroundings) and the same geometrical flow pattern (*i.e.* the boundary flow pattern) which is limiting in the main plane the subspace of flow pertaining to one particle. The liquid flowing through this subspace is said to flow around the considered particle in the main plane. The subspaces of flow are covering the main plane without voids and without the mutual overlapps (so called close coverage). *7*) If the origin of cylindrical coordinates is identical with the centre of the particle and if the positive direction of the axis is considered to be opposite to the direction of gravity force then the velocity field around the particles in the main plane is in points symmetrical with respect to the origin. Simultaneously it holds

$$v_r = v_\Theta = 0\,, \quad \text{if} \quad z = 0\,; \tag{1}$$

$$\frac{\partial^2 v_r}{\partial z^2} = 0\,, \quad \text{resp.} \quad v_z \frac{\partial v_\Theta}{\partial z} = \nu \frac{\partial^2 v_\Theta}{\partial z^2}\,, \quad \text{if} \quad z = 0\,; \tag{1a}$$

$$\frac{\partial^2 v_z}{\partial z^2} \neq 0\,, \quad \text{if} \quad r \leqq R\,, \quad z = 0\,; \tag{1b}$$

$$\mathbf{v} = \mathbf{v}(r)\,, \quad \text{if} \quad r \leqq R\,, \quad z = 0\,, \tag{1c}$$

where R is the radius of the circle inscribed into the boundary pattern (so called boundary circle). *8*) Stable can be only beds with such arrangement of particles in main planes at which the boundary circles of neighbouring particles are in contact. *9*) Spaces between boundary circles in the main plane are regions with the constant velocity $\boldsymbol{u}$, for which holds

$$\boldsymbol{u} = \mathbf{v}(R)_{z=0}\,, \tag{2}$$

i.e. the liquid velocity is constant and equal to the velocity in the circumference of the boundary circle. *10*) Stable can be only beds with arrangement of particles in the main plane at which the regions with constant velocity are mutually symmetrical in respect to the centre of that boundary circle with which they are in contact. *11)* If the conditions *1* to *10* hold, the forces equilibrium between the hydrodynamic, buyoancy and gravity forces (at zero horizontal component of hydrodynamic force in the main plane) is possible only for vertical arrangements of main planes according to one of the following principles: *a)* Main planes are arranged so that on the vertical line passing through the centre of any particle in some main plane is situated the centre of one particle in each next main plane (*i.e.* the particles are situated one above the other in vertical columns), *b)* centres of particles in any main plane are above the centres of symmetry of regions with constant velocity in the preceding main plane. *12)* Let us assume that such particles are found which form a uniformly fluidized bed for the given superficial liquid velocity. An infinite number of beds can be constructed from these particles which differ by the geometrical arrangement (but different from those for the uniformly fluidized bed) but in each of them *a)* the conditions *4* to *11* are met, *b)* at the considered liquid velocity on the particle the equilibrium of gravity, buoyancy and drag forces takes place.

All these beds are without rigid bonds unstable and are denoted as equivalent, unstably balanced beds (for the given velocity of the considered liquid and considered particles). The potential energy of the sufficiently large number of particles above the suitably chosen horizontal cross section in the uniformly fluidized bed is smaller than the potential energy of the same number of particles above the equally large horizontal cross-section in the arbitrary equivalent unstably balanced bed (at the given velocity of the considered liquid and for the considered particles).

Basic Assumptions

On the basis of experiments carried out, conditions *1* to *3* can be satisfactorily approached by a suitable choice of particles and of the liquid and by the suitable arrangement of the fluidized bed at $\mathrm{Ar} \leq 7{\cdot}2$.

The assumptions *5* and *6* are the simplest way how to state that on each particle in its stable equilibrium state the same hydrodynamic forces act and that liquid streams flowing around the neighbouring particles are mutually in contact. The boundary region of the subspace with the stream flowing around an arbitrary particle is also the boundary region of the subspaces with streams flowing around neighbouring particles.

Assumptions *7* to *10* are characterizing the geometrical arrangement of particles in main planes and are suplementary to condition *6*. At first, only part of cases is considered in which the condition *6* is satisfied for each particle in the main plane. Geometrically equal surroundings can be formed by other particles around each particle in the main plane *e.g.* in the case the distribution of particle is suitably related to uniform n-angles which can cover this area without voids and without overlaps. It is demonstrated that this condition is satisfied only by an equilateral triangle, a square and a hexagon. From planimetry it is known that the internal angle of the uniform n-angle is $\gamma = 180° - 360°/n$. The plane can be covered by uniform- n-angles so that first of all the region around each vertex of one considered n-angle is covered in this plane and then are covered in a similar way regions around free vertices of adjoining n-angles. Fig. 1 is characteristic for the first step where the region around the vertex, which is the intersection of sides a and b is covered at first. For the n-angles to cover completely the area between the arms of the angle $\omega + \gamma$ at one of their common vertices the condition must hold

$$360° = (k + 1)\gamma = (k + 1)(180° - 360°/n), \tag{3}$$

where k is the number of n-angles which must be assigned in the considered vertex to the first n-angle. If the condition (3) is met then it is possible to cover completely the area by the considered n-angles. The number of n-angles m which will be in mutual contact with the first one in Fig. 1 is given by the relation

$$m = k + (n - 2)(k - 1) + k - 2 = n(k - 1). \tag{4}$$

According to (3) it holds $k = (n + 2)/(n - 2)$ and after substitution into Eq. (4) the relation is obtained

$$m = 4n/(n - 2). \tag{5}$$

In agreement with the physical meaning k and $n > 2$ are integers. By substitution for k, the pairs are obtained: $k = 1$, $n = \infty$; $k = 2$, $n = 6$; $k = 3$, $n = 4$; $k = 4$ $n = 10/3$ (unsatisfactory); $k = 5$, $n = 3$. With increasing value $k > 5$ the value of $n < 3$ is monotonously decreasing but $n < 3$ has no physical meaning. This means that among all equilateral n-angles the area can be closely covered only by equilateral triangles, squares and equilateral hexagons. A close cover by equilateral hexagons can be considered as a suplementary specification in the net of close coverage by equilateral triangles. From Eq. (5) results that for the closely covered area,

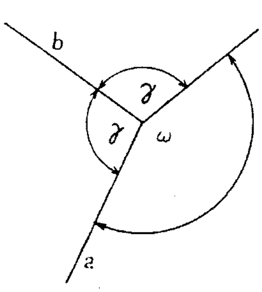

FIG. 1
Procedure for Close Coverage of the Plane by Equal Regular Polygons

the equilateral triangle has twelve neighbouring tringales, the square has eight adjacent squares and the equilateral hexagon has six neighbouring hexagons.

Limitations set by condition *6* to the boundary surroundings and boundary flow pattern is the application of condition *5*, to the main plane. According to condition *5* a close relation must exist between the geometries of the boundary surroundings and the boundary flow pattern. Both of them are determining the flow pattern around the particle in the main plane by the stream which is adjoined to it up to limitations in the contact with the same streams around neighbouring particles.

In accordance with the considerations on close coverage by equilateral *n*-angles condition *6*. is fulfilled if particles are situated by their centres in corners of squares which are closely covering the main plane. In this case it is possible to imagine two arrangements of boundary flow patterns which satisfy simultaneously the assumption *5*:

a) The boundary flow patterns (dashed line) have a shape of squares in the centre of which are particles, see Fig. 2.

b) The boundary flow patterns have the shape of isosceles triangles with situation of particles according to Fig. 3. It can be seen from this Fig. that a close coverage of the area by isosceles triangles is again the coverage by squares defined in another way.

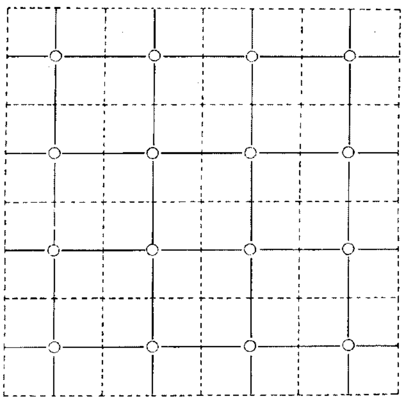

FIG. 2

Boundary Surroundings of Particles and Boundary Flow Patterns when the Particles Are Situated in the Corners of Squares and the Boundary Flow Pattern Is a Square

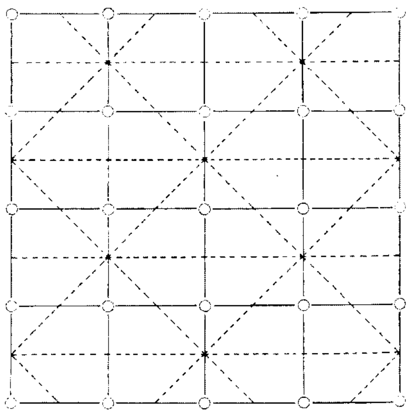

FIG. 3

Boundary Surroundings of Particles and Boundary Flow Patterns when the Particles Are Situated in Corners of Squares and the Boundary Flow Pattern Is a Isosceles Triangle

In Fig. 4 there is the main plane closely covered by equilateral triangles (dashed mesh) and on their basis is additionally constructed by solid lines the mesh of equilateral hexagons. Positioning of particles and of boundary flow patterns can be made according to Fig. 4 – in accordance with assumption *6*, at the simultaneous validity of condition *5* – in two ways:

a) Centres of particles are situated in the corners of hexagons drawn in full lines; in this case the boundary flow patterns represent equal equilateral triangles in the main plane.

b) Centres of particles are situated in the corners of triangles drawn in dashed lines; in this case the boundary flow patterns represent in the main plane equal regular hexagons.

The condition for pattern of particle distribution and liquid velocity in the main plane to be symmetrical and equal with respect to the centre of any particle is positioning of centres of particles in the centre of symmetry of the boundary flow pattern. In such case the zero horizontal component is the resultant of forces by which other particles in the main plane act *via* liquid on an arbitrary particle. As is demonstrated in the following part, the zero value of the horizontal component of this resultant is one of conditions of existence of a force equilibrium in the bed. The problem of the velocity field around the particle in the main plane, according to Figs 2 to 4 is partially a problem of flow around a perfectly isometric body in the plane limited

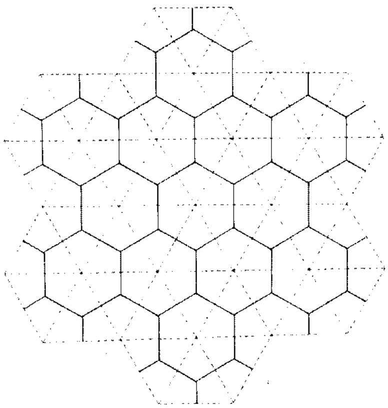

FIG. 4

Mesh for Distribution of Particles and Limitation of Boundary Flow Patterns in the Main Plane by Use of the Equilateral Triangle and Equilateral Hexagon

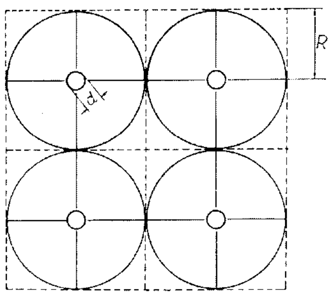

FIG. 5

Boundary Circles in Square Boundary Flow Patterns by an equilateral n-angle where n is a relatively small number. This fact is represented by assumption 7, which has to be considered as a heuristic hypothesis of the theoretical model. The velocity field has a circular symmetry in the main plane up to some distance R from the centre of particle. The circular symmetry in the whole region limited by the boundary flow pattern seems improbable. The requirements according to which the boundary circle has to be inscribed into the boundary patterns has a broader physical meaning which appears when into the mesh according to Figs 2 and 3 the boundary circles are drawn. To Fig. 2 then corresponds Fig. 5 and the boundary circle inscribed into the triangle in Fig. 3 can be imagined. The circular symmetry of the velocity field is due to the particle from which it spreads up to the edge of the boundary pattern where its progress is stopped by an equally strong effect of the adjacent particle in the main plane, so that the boundary circles around adjacent particles must be in contact similarly as those in Fig. 5. This is also included in the assumption *8*. which is obviously of importance only if the boundary circles are inscribed into the boundary patterns. Unlike in Fig. 5 the boundary circles inscribed into triangles in Fig. 3 cannot be in contact and this arrangement must be considered as unrealistic.

The boundary circles in Figs 6 and 7 correspond according to the above proposed model to the mesh in Fig. 4. It can be proved that if into centre of each hexagon in Fig. 6 is situated the same circle as those in its corners the arrangement according to Fig. 7 is obtained. The continuous transfer from the velocity of hindered settling

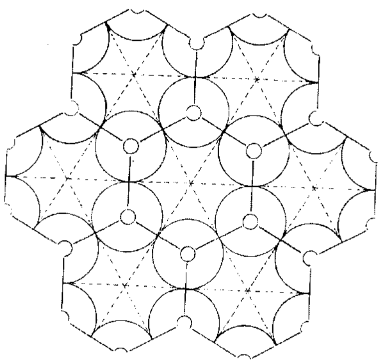

FIG. 6

Boundary Surroundings of Particles and Distribution of the Main-Plane to Subspaces of Flow with Boundary Circles when the Boundary Flow Patterns Are Equilateral Triangles and the Centres of Particles Are in Corners of Equilateral Hexagons

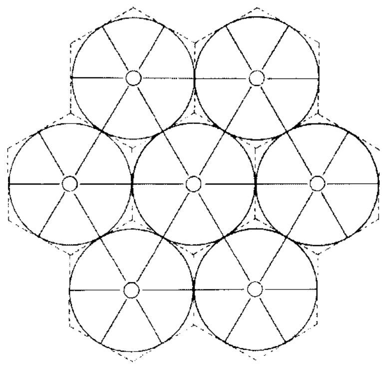

FIG. 7

Boundary Surroundings of Particles and Distribution of the Main Plane to Subspaces with Boundary Circles when the Flow Patterns Are Equilateral Hexagons and Particles Are Situated in Corners of Equilateral Triangles of the suspension at $\varepsilon \to 1$ to the velocity of free settling of an individual particle in an infinite liquid justifies the assumption for the bed the validity of the relations (*1*) to (*1b*) because the equations of this type are valid also for creeping flow around individual particles.

The voids between the boundary circles in Fig 5 to 7 resemble the outlet hole from a vessel wall as there mouth streamlines symmetrically curved into oposing sides. The velocity has a constant value on the whole perimeter of the void and the circular symmetry of the field ends there. This leads to the simplest model consideration of the velocity field in the voids which is heuristicaly formulated by the hypothesis *9*.

The survey of all geometrical structures in the main plane which comply with assumptions *5*) to *8*) can be obtained from the following consideration: Let us define a number of circles as a system of mutually contacting circles with their centres situated on a straight line (so-called axis of a row). The rows of circles are arranged in the main plane in parallel so that the circles of two adjacent rows are in contact. The mutual contact of circles of two neighbouring rows is satisfying condition *8* which requires that the axis of a given row and the straight line connecting the centre of a circle in this row with the centre of another circle (from the adjacent row) with which it is in contact, form an angle α from the range $\alpha \in \langle \frac{1}{3}\pi, \frac{1}{2}\pi \rangle$. This means that conditions *5* to *8* require that the particles on the main plane be situated in corners of squares ($\alpha = \frac{1}{2}\pi$) and rhombs with a sharp angle $\alpha \in \langle \frac{1}{3}\pi, \frac{1}{2}\pi \rangle$. The equilateral triangles and hexagons can be considered as a sufficient specification in the mesh of rhombs with a sharp angle $\alpha = \frac{1}{3}\pi$. For the angle $\alpha \in \langle \frac{1}{3}\pi, \frac{1}{2}\pi \rangle$ the conditions *5*

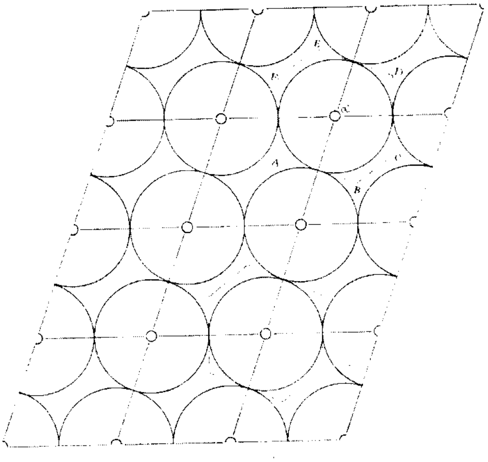

FIG. 8

Situation of Rows of Boundary Circles in the Main Plane at Their Shift by the Same Angle α in the Same Direction. The Hexagon ABCDEF Represents the Boundary Flow Pattern. Particles Must Be Situated in Corners of Rhombs to *8* can be met in four ways (here condition *5* is definitely met at least in the main plane):

a) Rows of circles are in the main plane situated so that each consecuting is in respect to the preceding one shifted in the same direction by an equal angle α, see Fig. 8.

b) Rows of circles in the main plane are situated according to the following rule: If an arbitrary row is chosen as the zero and the next rows, are numbered 1, 2, 3, ... then an arbitrary row $2n + 1$ (where $n = 1, 2, 3...$) is shifted in the same direction in respect to the $2n$-th row by the same angle α and an arbitrary $2n$-th row is shifted in the same direction in respect to the $2n - 1$ row by the same angle β, where $\alpha \neq \beta$, but $\alpha \in \langle \frac{1}{3}\pi, \frac{1}{2}\pi \rangle$, $\beta \in \langle \frac{1}{3}\pi, \frac{1}{2}\pi \rangle$. Fig. 9 gives one of these cases.

c) Row of circles situated in the main plane is alternately shifted in respect to the preceding one by the same angle α so that behind the row shifted to right side is situated the row shifted toward the left side *etc.* see Fig. 10.

d) Rows of circles which are situated one behind the other in the main plane are mutually shifted in opposite directions at two different angles, *i.e.* behind the row which is in respect to the last one shifted toward the right side by the angle α is situated the row which is in respect to it shifted toward the left side by the angle β; the next one is again shifted toward the right side by the angle α *etc.* Such arrangement is demonstrated in Fig. 11 for $\alpha < \beta$.

By positioning particles in centres of boundary circles arranged according to Figs 6, 9 to 11 the layers are obtained where in the main plane the distribution of particles

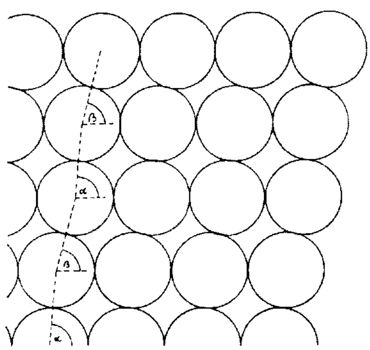

FIG. 9

Situation of Rows of Boundary Circles in the Main Plane at Alternately Varying Angle [α, β] of the Shift in the Same Direction

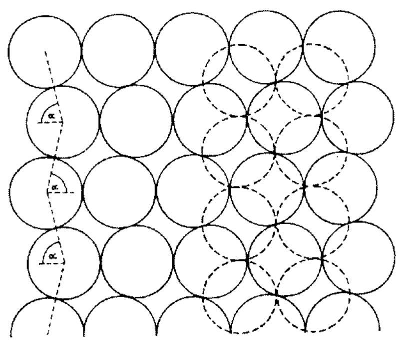

FIG. 10

Situation of Rows of Boundary Circles in Main Plane at Constant Angle α and Alternately Varying Direction of the Shift. Rows of Circles in Neighbouring Main Planes Are Dashed and the liquid velocity is not symmetrical as to the centre of particles. This is because the regions with the constant velocity situated around the boundary circle are not mutually symmetrical according to its centre. According to assumption *10.* the beds with such distribution of particles in the main plane cannot be stable unlike of those beds with symmetrical distribution according to Figs 5, 7 and 8. The physical meaning of assumption *10* can be understood from the following consideration: From condition *5* results that the neighbouring main planes cannot be positioned arbitrarily, *i.e.* that between the arrangement of particles in the main plane and the vertical positioning of main planes exists relation which reversely affects admissibility of particle distribution in the main plane. A typical example of arrangement of main planes contradictory to condition *5* is given in Fig. 12. The angle α represents rotary shift of boundary flow patterns in the second main plane related to the boundary pattern in the first main plane which is situated below.

Condition *5* is satisfied at two types of arrangements of main planes which are given by the assumption *11*. It seems that at conditions *5* to *9* these are the only satisfactory arrangements of main planes. Arrangement according to assumption *11a* is denoted as the structure of type *A* and the arrangement according to assumption *11b* as the structure of type *B*. For the structures of type *A*, condition *5* is satisfied in the space for all the beds where it is satisfied in the main plane. This means that in accordance with condition *5*, there is formally no difference between

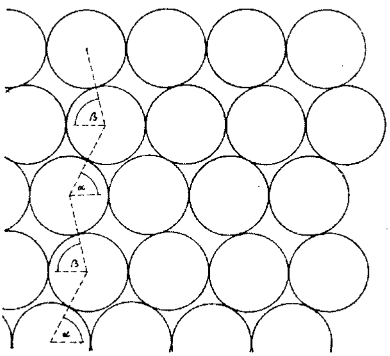

FIG. 11

Situation of Rows of Boundary Circles in the Main Plane at Alternately Varying Shifts by the Angle α to the Right and by the Angle β to the Left

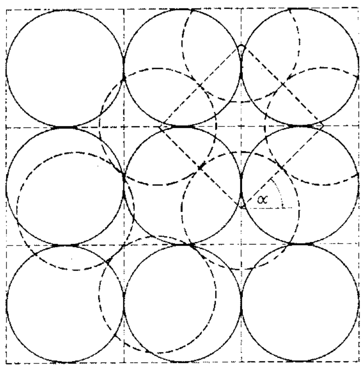

FIG. 12

Example of Vertical Arrangement of Two Main Planes which Is in Disagreement with the Assumption that the Flow Pattern at Flow Around Each Particle in the Bed Is the Same. Centres of Particles (not indicated) Are in the Corners of Squares. Boundary Patterns are Squares the symmetrical and asymmetrical arrangement in the main plane. But we argue the assymetric structures in Figs 6, 9 to 11 as follows: Let us imagine a duct between two adjacent main planes which is formed by elementary streamfilaments which pass through regions with a constant velocity situated one behind the other. In this vertical duct are elementary streamfilaments spreading at the bottom and narrowing toward the top. If the distance of main planes is h then the increment of the specific pressure energy on the distance h is given by the relation

$$\Delta e_{\mathrm{p}} = -[\boldsymbol{g} h \varrho_{\mathrm{f}} + \boldsymbol{g}(\varrho_{\mathrm{s}} - \varrho_{\mathrm{f}})(1 - \varepsilon) h]/\varrho_{\mathrm{f}} . \qquad (6)$$

From this and from the properties of flow in the region with constant velocity results that the mean specific dissipated energy in such a duct equals to

$$e_{\mathrm{diss}} = [\boldsymbol{g}(\varrho_{\mathrm{s}} - \varrho_{\mathrm{f}})(1 - \varepsilon) h]/\varrho_{\mathrm{s}} \qquad (7)$$

and thus flow in the duct is not potential. There exist viscous stresses which are dependent on the shape, orientation and size of the region with the constant fluid velocity. These stresses are partially characterizing the velocity field between two main planes and thus also the planary forces on the surface of the particle. Let us assume that the particles are arranged in the horizontal main plane according to one of symmetrical structures given in Figs 5, 7 or 8. Through the system of particles so arranged in an isolated plane the fluid is flowing so that at a sufficient distance from the particle the velocity field is homogeneous, the velocity vector is perpendicular to the main plane and the change in velocity is the result of action of particles only. It is obvious that the elementary surface forces act on the particle in pairs for which the horizontal components are canceled as the velocity field in the subspace ascribed to the particle by the stream in an arbitrary plane parallel with the main plane has a point symmetry in respect to the intersection of the considered plane with the vertical passing through the centre of particle. In assymetrical arrangements which are given in Figs 6, 9 to 11 the prior point symmetry of the velocity field in an arbitrary horizontal plane is upset in the subspace with the stream ascribed to the particle. The horizontal components of elementary surface forces on the particle are not cancelling each other in the pairs. If the stream around the particle in an isolated main plane has a symmetry in which the horizontal components of elementary surface forces on the particle cancel each other, then for the force equilibria must suffice to arrange the main planes so that this symmetry is not upset and their presence affects only the vertical component of elementary surface forces on the particle. If the stream around the particle in an isolated main plane does not have a symmetry necessary for cancelation of the horizontal components of elementary surface forces, the layers in main planes must be arranged for the force equilibria so that their presence affects simultaneously the vertical as well as horizontal component of elementary surface forces on the particle. It must be expected that in general from the condition of zero horizontal component of the resultant of surface forces another arrangement of main planes results than from the condition of force equilibria for the gravity, buoyancy, and vertical component of the resultant of surface forces on the particle. Let, however, us assume that the force equilibrium can exist also at the asymmetric arrangement of particles in main planes. Disturbance moving the particles toward the narrower ducts, would then cause a greater drag than at their shift toward the wider ducts and it seems that such system would finally form a symmetrical structure.

These considerations support hypothesis *10.* which should be also valid for structures of both types *A* and *B*.

Condition *5* for the structures of type *B* obviously cannot be met for asymetric arrangements given in Figs 6, 9 to 11. Also the structures of type *B* according to Fig. 8 are probable. Here, it seems improbable, that after hitting the particle, the liquid from a narrow stream will form a circularly symmetrical region in the boundary circle on the periphery of particle.

It results from the above that the structure of type A can have particles arranged in the main plane according to Fig. 5 (structures of type A_1) according to Fig. 7 (structures of type A_2) and according to Fig. 8 (structures of type A_x) while the structures of type *B* can have the arrangement according to Fig. 5 (structures of type B_1) or according to Fig. 7 (structures of type B_2 or B_3).

For particles arranged in the main plane according to Fig. 7 for structures of type *B*, the main planes can be situated according to Figs 13 or 14. For the main plane denoted as zero then *a*) for arrangement of particles according to Fig. 13 (structures

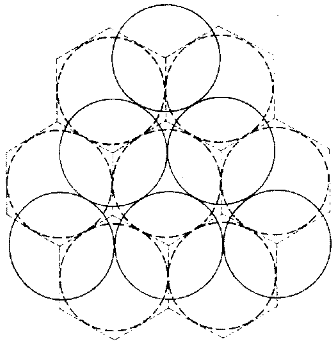

FIG. 13

Mutual Situation of Main Planes for Structure of the Type B_2

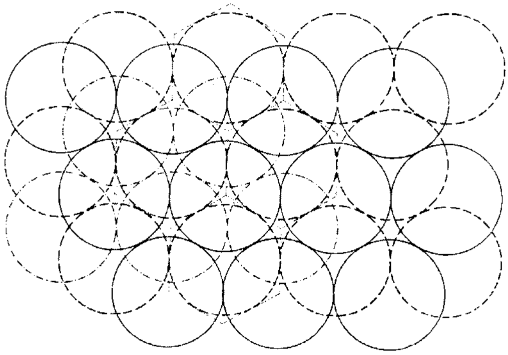

FIG. 14

Mutual Situation of Main Planes for Structure of the Type B_3

of type B_2) their centres in an arbitrary $2k$ main plane (where k is an integer) are situated exactly above the particle centres at the zero main plane and the centres of particles in the first and each $(2k + 1)$th plane lay exactly above the centre of symmetry of the half number of regions with a constant velocity in the zero plane. Along the whole bed height three straight ducts are situated around each particle the horizontal cross-sections of which change in alternately their orientation in the main planes. The streams from other three regions with a constant velocity are hitting the particles in the adjacent main plane. Distribution of these streams and of direct ducts on the circumference of the boundary circle is not symmetrical with respect to its centre, but the over-all distribution of hitting streams and of direct ducts is symmetrical in respect to the three vertical planes which are passing through the centre of particle. With respect to these planes we can assume that for the force equilibria the horizontal components of the resultant by which other particles have acted on a particle in the main plane is nill. For these reasons this arrangement has been considered possible.

b) For an arbitrary arrangement according to Fig. 14 (structures of the type B_3) centres of particles in an arbitrary $3k$-th main plane are situated exactly above the centres of particles in the zero main plane; centres of particles in the first and each $(3k + 1)$th main plane are situated exactly above the centres of symmetry of the half number of regions with a constant velocity in the zero main plane. Centres of particles in the second and in the each $(3k + 2)$th main plane are situated exactly above the centres of symmetry of the second half of regions with a constant velocity in the zero main plane. Around each particle at a certain main plane, there are three straight vertical ducts which originate in the adjacent lower main plane and terminate by the direct hitting the particles in the next higher main plane. Other three ducts around the particle are formed in the considered main plane which are passing at the varied orientation through the first adjacent main plane and are terminating by the direct hit on particles in the second higher main plane. From condition 5 results that the distance between the zero and first main plane is the same as between the first and second main plane. In each fourth main plane the distribution of particles is exactly repeated.

Distance of Main Planes

From the geometry results, that quantities-particle diameter d, radius of the boundary circle R, distance of main planes h and porosity of the bed ε are at structures of type A and B bound by the relation.

$$F(d, h, R, \varepsilon) = 0\,, \tag{8}$$

where the form of the function F can be determined for each assumed arrangement of particles. Since in each main plane is the arbitrary particle surrounded by other particles symmetrically to its centre, the hydrodynamic effect of these particles can cause only an additional force in the vertical direction. As is known[1], this force increases the drag force in comparison with that which would be found at the same superficial velocity for an individual particle. From the experiments[1] results, that at the same structures A the layers above and below the considered main plane are causing decrease in the drag force. For the same distance of main planes from the considered main plane the drag force is more reduced by the layer preceding the considered main plane. For structures of the *B*, the bed in an arbitrary main plane is causing a considerable increase of the drag force on particles in the next following main plane in the direction of flow as it is increasing the local liquid velocity around the particles. If for structures of the type *A* or *B* the constant distance of particles $2R$ is kept in the main plane and the distance of main planes is varied, only vertical components of elementary forces on the surface of particles do change at constant velocity of the chosen liquid, and their horizontal components are cancelling each other in pairs as the symmetry of flow around the particle is not changing. From nature of the effect of surrounding particles on the drag force given here directly results the following important conclusion: „For an arbitrary structure of the type A or B there exists an interval of velocities in which to a certain definite velocity corresponds a sequence of mutually corresponding pairs of quantities h and R from continuous intervals (and thus probably also a continuous interval of porosities ε) at which the equilibrium of gravity, buoyancy and drag forces on the particle takes place," It is possible to imagine a fluidized bed in which the particle distribution is randomly varied at a constant superficial liquid velocity so that the equilibrium of forces on the particle is not considerably affected *i.e.* that the pairs of quantities h and R exist (and to them corresponding value ε) at least in part of intervals which at constant superficial velocity are limiting the equilibrium of forces. In gravity field the potential energy of particles in such a bed should vary and would be accompanied by pressure and velocity pulsations. Such consideration would be acceptable in agreement with the properties of the bed at larger values of the Ar number. At small values it is more exact to accept the assumption *1* according to which at constant superficial liquid velocity both the porosity and position of particles in the bed are constant. This means that under such conditions for given particles the uniformly fluidized bed will have at some liquid velocity only one porosity with fixed values of h and R for the given type of structure. To all other pairs of values h and R from a certain structure of the same type and from structures of other types at which the equilibrium of forces can exist, corresponds an unstable arrangement of particles in beds without rigid bounds. The condition of stability is considered in the next part of this study. On basis of hydrodynamic considerations and stability conditions for a uniformly fluidized bed it should be possible to derive a relation for h/R independent of Eq. (*8*). With respect to complexity of the problem analytical solution of such problem has not been looked for. It was possible to find by a semi-empirical method quantities on which the ratio h/R depends. Let us assume that the relation holds

$$h/R = \varphi(\pi_1, \pi_2, \ldots)\,, \tag{9}$$

where φ is an unknown function of prior unknown parameters $\pi_1, \pi_2 \ldots$. If on basis of the theoretical model which has been characterized earlier (and into which is also included Eq. (9)) the mathematical model is proposed, solved and compared with the experiment (which is also demonstrated in the remaining part of this study) it is obtained that for Ar $\leqq$ 7·2 Eq. (9) must have the form

$$h/R = \varphi(\varepsilon)\,. \tag{10}$$

If the validity of the model would be extended to region of Ar > 7·2 then it should hold

$$h/R = \varphi_y(\mathrm{Ar}, \varepsilon)\,. \tag{11}$$

Instead of Eq. (11) it is possible to use other equations, *e.g.* $h/R = \varphi_{y1}(\mathrm{Re}_t, \varepsilon)$, $h/R = \varphi_{y2}(\mathrm{Ly}, \varepsilon)$ as the Ar and Re_t or Ly numbers are bound by a unique relation.

From Eqs (8) and (10) results that for the uniformly fluidized bed at Ar $\leqq$ 7·2 a physically correct relations could be found, which are of the types

$$\left.\begin{aligned} R &= dF_1[\varepsilon, \varphi(\varepsilon)] \\ \text{or} \qquad h &= dF_2[\varepsilon, \varphi(\varepsilon)] \end{aligned}\right\}. \tag{12}$$

Functions $F_1(\varepsilon)$ and $F_2(\varepsilon)$ are known as the function F in Eq. (8) is known, but function φ must be determined experimentally by use of the mathematical model.

First Condition of Stability of Beds with Structures of A *and* B *Types*

By fixing the position of particles by rigid bonds it is possible to prepare beds with structures of all considered types A and B for all pairs of quantities h and R at which the equilibrium of forces takes place. By fixing the required superficial liquid velocity, the state with the forces in equilibrium at small Ar numbers is reached. If such arbitrary bed is formed which at the force equilibrium has a different particle distribution than the uniformly fluidized bed (such beds have been for condition *12* denoted as equivalent unstably balanced) and the solid bonds are removed a spontaneous particle motion takes place which leads to the formation of the uniformly fluidized bed with the porosity fully corresponding to the given liquid velocity. The reason of this should be seen as follows: for the creeping flow around the particle there also appear in the liquid, due to non-uniformity of particle surface and other reasons, small disturbance which disappear with time. Due to this, the equilibrium of forces on the particle is slightly disturbed and particles are displaced from the equilibrium position. These displacements are the first step for decay of unstable structures. The attempts to solve the problem of stability of structures A and B in the frame of analytical mechanics from this point of view have not yet been succesful. Spontaneous transfer of arrangements in unstable equilibrium to those in stable equilibrium of the uniformly fluidized bed is an irreversible process related with the energy dissipation at overcomming the drag of the liquid by the moving particles. From the given analysis is obvious that it is possible to reach also reversibly from an arbitrary state the state of stable equilibrium at constant superficial liquid velocity if the particles are moving by an infinitesimal velocity only on such trajectories which correspond to the transitional equilibrium states. Such trajectories between two arbitrary states in equilibrium are possible as the intervals of values h and R, the pairs of which are determining the equilibrium states, are continuous and to an infinitesimal change in h corresponds an infinitesimal change in R if for a new arrangement of particles the equilibrium of forces is reached. If the potential energy (in the earth gravitational field) of particles in the initial state of unstable equilibrium differs from the potential energy in the final state of stable equilibrium then for the reversible transfer in the environment a reversible work must be done in which the potential energy of particles is consumed if its value in the bed decreases, or the potential energy is added to the particles if its value in the bed increases. If the potential energy of particles in the state of stable equilibrium were greater than the potential energy of particles in any of unstably balanced states the spontaneous transfer could not take place as the impulse of forces resulting from small disturbances is accidentally changing its direction and their power output is small in comparison with the power output of forces which are causing the energy dissipation and which are necessary for an increase in the potential energy of particles. At a spontaneous transfer from the state with a larger potential energy to the states with a smaller potential energy the over-all loss in potential energy is dissipated. The contribution of the power output of forces resulting from small disturbances can be at such a spontaneous transfer negligible.

If thus at a constant superficial liquid velocity at least some of particle arrangements according to structures of types A or B, at which the equilibrium of forces takes place, differ by the potential energy of particles then according to the aforesaid, assumption *12* should hold as a very probable hypothesis. From the next consideration results a simple physical meaning of this hypothesis: Let us consider the potential energy of particles in the uniformly fluidized bed and in beds which are with it equivalent unstably balanced. As the basical horizontal cross-section situated in the bed at the level $z = z_0 = 0$ we choose a uniformly widened boundary pattern which is characteristic for the just considered type of structure. If the type of structure is altered, the shape of the basical cross-section will also change but its area will remain constant. If this cross-section is oriented in the horizontal plane so that its sides are parallel with sides of boundary patterns, the formal assignment of particles to those regions of the basical cross-section on its perimeter which are not fully covering the boundary pattern is simplified. Let us assume that the area of the basical cross-section S is sufficiently large so that the number of the formally assigned particles would be negligible in comparison with the total number of particles N_1 which are in the main plane on the area equal to the area of the basical cross-section. The vertical planes situated in edges of the basical cross-section form with this cross-section as the base an angular vessel open at the top. Part of main planes which are situated in this vessel contain the equal number of particles N_1. If the boundary pattern limits the area S_1 on the main plane then it holds

$$N_1 = S/S_1 \,. \tag{13}$$

Let us consider in this angular vessel the mean specific potential energy of a sufficiently large constant number of particles N. This mean specific potential energy is changing at the transfer from the structure of the uniformly fluidized bed to different arrangements of particles in equivalent unstably balanced beds. If the arrangement of particles in the bed is known, it is possible to calculate the volume of the bed V_1 which corresponds to one particle, and it obviously holds

$$z = NV_1/S \,, \tag{14}$$

where z is the height of the bed assigned to the considered N particles in the angular vessel. Let us assume that the basical cross-section is situated in the middle between two main planes on the level $z_0 = 0$ to which the zero specific potential energy is assigned. Centres of gravity of N_1 particles in the lowest main plane in the limited volume of the bed have in respect to $z_0 = 0$ the position $z_1 = h/2$. Remaining $N - N_1$ particles are forming formally q layers, where

$$q = (N - N_1)/N_1 \,. \tag{15}$$

The number q can be written as the sum

$$q = x + y \,, \tag{15a}$$

where x is an integer part of number q *i.e.* $x = [q]$ and for the decimal part y holds $y \in \langle 0; 1)$. If $y = 0$ then $z = (x + 1)h$ and the angular vessel is intersected by $x + 1$ main planes. First of them is in the distance from $z_0 = 0$ for $h/2$ and the level z is in the middle between the $x + 1$ and $x + 2$ main planes. The over-all potential energy

E_z of considered particles is then, according to the definition, equal to

$$E_z = N_1 \pi g d^3 \varrho_s (x + 1)^2 h/12 . \qquad (16)$$

If $y \in (0; 1)$, then in a limited volume the N considered particles form beside the integer $x + 1$ sections of main planes part of the $(x + 2)$th section in which they formally participate by yN_1 particles. Potential height of these yN_1 particles is $(h/2) + (x + 1) h$. Instead of Eq. (*16*) it should be written

$$E_z = N_1 \pi g d^3 \varrho_s [(x + 1)^2 h + y(2x + 3) h]/12 . \qquad (17)$$

If Eq. (*17*) is divided by the term $\pi N d^3 \varrho_s/6$, the relation is obtained for the mean specific potential energy e_z of particles related to the considered constant number of N particles in the angular vessel with the height z. If into the so arranged equation the following substitutions are made: for x the relation $q - y$ from Eq. (*15a*), for q the right hand side of Eq. (*15*), for N from Eq. (*14*), and for N_1 from Eq. (*13*), the relation is obtained

$$e_z = (g V_1/2zS_1) [(zS/V_1)^2 h - y(y - 1) h] . \qquad (18)$$

In the next part is demonstrated that for structures of all A and B types holds $V_1 = hS_1$ and thus from Eq. (*18*) is obtained

$$e_z = (gh/2z) [(z^2/h) - y(y - 1) h] . \qquad (19)$$

At a sufficiently large number of particles N which are suitably chosen $z^2/h \gg y(y - 1) h$ and it is possible to write

$$e_z \approx gz/2 . \qquad (20)$$

From this and from assumption *12* results that if above a sufficiently large cross-sectional area a uniformly fluidized bed and all with it equivalent unstably balanced beds are constructed at the given liquid velocity from the sufficiently large number of particles the uniformly fluidized bed in between them would have the smallest potential energy of particles and thus the smallest bed height and also the smallest mean volume of the bed which corresponds to one particle. This is the principal physical meaning of assumption *12*.

It is necessary to say that for the sake of accuracy, the potential energy of particle should be considered in the form $g\pi d^3(\varrho_s - \varrho_f) z/6$ instead of the earlier used relation $g\pi d^3 \varrho_s z/6$. But this difference has no effect on the use of e_z as the criterion of stability. Hypothesis *12* is denoted as the first condition of stability of beds with structures of the A and B types. By this we want to stress the experimentally known fact that there exist also some other conditions which must be fulfilled so that the uniformly fluidized (stable) bed could exist.

The proposed mathematical model is considered as one of the simplest yet proposed. It enables to formulate the hydrodynamic problem of the uniformly fluidized bed, to form the mathematical model and obtain a significant solution. By formulating anew the assumption 7 which is the sensitive part of the model, it is possible to construct more complicated mathematical models with other assumptions remaining valid. *E.g.* the boundary circle can be deformed (at once or in dependence on porosity) symmetrically in respect to its centre so that analogy of condition *8* be fulfilled according to which the adjacent so deformed sections are in contact. But this means that it is necessary to change assumptions given by Eqs (*1*) and (*1c*) and reject the assumption of circular symmetry of the velocity field in the region of the subspace limited by the deformed boundary circle or to reject the assumption that the velocity field is homogeneous in spaces between the deformed boundary circles.

LIST OF SYMBOLS

a, b	adjacent sides of the uniform n-angle
$\mathrm{Ar} = gd^3(\varrho_s - \varrho_f)\,\varrho_f/\mu^2$	Archimedes number
d	diameter of a spherical particle
e_p	specific pressure energy of liquid
e_{diss}	specific dissipated energy of liquid in section h
e_z	mean specific potential energy of particles according to (*18*)
E_z	potential energy of particles according to (*16*)
F, F_1, F_2	functions
g	gravitational acceleration
h	distance of main planes in the bed
k	number of uniform and equal n-angles which have to be assigned to one vertex of such n-angle at a close coverage of the plane
$\mathrm{Ly} = \mathrm{Re}_t^3/\mathrm{Ar}$	Ljashchenko number
m	number of regular and equal n-angles which are surrounding at the closely coverage of the plane the arbitrary n-angle in this plane
n	number of angles in a regular polygon
N	number of particles in the bed above the chosen cross-section to which the potential energy in the terrestrial gravitational field is related
N_1	number of particles which are in the part of main plane limited by the chosen cross-section S at determination of potential energy of particles in the bed
q	number of layers (main planes) according to Eq. (*15*)
r	radial cylindrical coordinate, when the origin of the system is in the particle centre
R	radius of the boundary circle (inscribed into the boundary flow pattern)
$\mathrm{Re}_t = dv_t\varrho_f/\mu$	Reynolds number at velocity of free settling
S	area of basical cross-section at determination of the potential energy of particles in the bed
S_1	cross-sectional area of boundary pattern

u	fluid velocity on the boundary circle and in the region of constant velocity
v_r	component of local liquid velocity in the direction of the coordinate r at points of the boundary flow pattern in the main plane
v_t	free settling velocity of particles in the actual fluid
$\mathbf{v}_z = \mathbf{v}_z(r)$	local fluid velocity in points of the main plane around the particle at $r \leqq R$
v_Θ	component of local fluid velocity in the direction of cylindrical (angular) coordinate Θ in points of boundary flow pattern in the main plane
V_1	volume of the bed corresponding to one particle
x	integer part of number q according to Eq. (*15a*)
y	decimal part of number q according to (*15a*)
[ ]	integer part of the number in the text
α, β	angles according to Figs 8 and 9
γ	angle in regular polygon, Fig. 1
Δe_p	increment in specific pressure energy of fluid in duct of a bed in the section h
ε	porosity of the uniformly fluidized bed or of a bed with some other structure of the type A or B
$\pi_1, \pi_2, \ldots$	dimensionless quantities
ϱ_f	density of fluid
ϱ_s	density of particles
$\varphi(\varepsilon)$	quantity determining the ratio h/R for the structure of the given type and for a given arrangement of particles
ω	angle according to Fig. 1.

REFERENCES

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